"In presenting the dissertation as a partial fulfillment of the requirements for an advanced degree from the Georgia Institute of Technology, I agree that the Library of the Institution shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to copy from, or to publish from, this dissertation may be granted by the professor under whose direction it was written, or, in his absence, by the dean of the Graduate Division when such copying or publication is solely for scholarly purposes and does not involve potential financial gain. It is understood that any copying from, or publication of, this dissertation which involves potential financial gain will not be allowed without written permission.

# THE OPTIMUM DESIGN OF LOW POWER MECHANICAL DRIVES FOR SERVOMECHANISMS

A THESIS

Presented to

the Faculty of the Graduate Division

Georgia Institute of Technology

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Mechanical Engineering

By

Albert Louis Holliman

December 1956

THE OPTIMUM DESIGN OF LOW POWER

MECHANICAL DRIVES FOR SERVOMECHANISMS

Approved:

Date Approved by Chairman: Dec. 13, 1956

TABLE OF CONTENTS

# LIST OF TABLES

# LIST OF ILLUSTRATIONS

## LIST OF SYMBOLS

$I_{AA}$ = moment of inertia of a body referred to its axis of symmetry (oz-in.$^2$)

$I_{BB}$ = moment of inertia of a body referred to an axis parallel to its axis of symmetry (oz-in.$^2$)

$D_o$ = outside diameter of cylinder (in.)

$D_i$ = inside diameter of cylinder (in.)

t = thickness of cylinder (in.)

d = density of material of cylinder (oz/in.$^3$), (Table 1)

$X_o$ = perpendicular distance from the axis of symmetry of the cylinder to a parallel axis (in.)

$D_p$ = pitch diameter of pulley, drum, or gear (in.)

W = weight of body or rack (oz)

D = pitch diameter of gear (in.)

n = number of teeth on gear

P = diametral pitch of gear

$n_1$, $n_2$, $n_3$, $n_4$ = number of teeth on gear 1, 2, 3, 4, respectively

$d_1$, $d_2$, $d_3$, $d_4$ = density of gear 1, 2, 3, 4, respectively (oz/in.$^3$)

$D_1$, $D_2$, $D_3$, $D_4$ = pitch diameter of gear 1, 2, 3, 4, respectively (in.)

$t_1$, $t_2$, $t_3$, $t_4$ = thickness of gear 1, 2, 3, 4, respectively (in.)

$K_A$, $K_B$ = constants for Mesh A, B, respectively (varies with the density and face width of the pinion and gear of a mesh)

$R$ = overall gear ratio between motor and load (greater than one)

$R_A$, $R_B$ = gear ratio of Mesh A, B, respectively

$T_M$ = motor output torque (in.-oz)

$T_L$ = load friction torque (in.-oz)

$T_{LM}$ = load torque referred to motor shaft (in.-oz)

$N_M$, $N_L$ = speed of motor and load, respectively (rpm)

$I_M$, $I_L$ = inertia of motor and load, respectively (oz-in.-sec$^2$)

$I_1$, $I_2$, $I_3$, $I_4$ = inertia of gear 1, 2, 3, 4, respectively (oz-in.-sec$^2$)

$I_{LM}$ = load inertia referred to motor shaft (oz-in.-sec$^2$)

$I_{2M}$, $I_{3M}$, $I_{4M}$ = inertia of gear 2, 3, 4, respectively, referred to motor shaft (oz-in.-sec$^2$)

$I_{23}$ = inertia of jack shaft between gear 2 and gear 3 (oz-in.-sec$^2$)

$I_{23M}$ = inertia $I_{23}$ referred to motor shaft (oz-in.-sec$^2$)

$\alpha_L$ = load acceleration (rad/sec$^2$)

$\alpha_M$ = motor acceleration (rad/sec$^2$)

$T_{MF}$ = motor friction torque (in.-oz)

$B$ = total linear backlash of a meshed pair of gears measured on pitch circle (in.)

$T_s$ = width of tooth space on pitch circle (in.)

$T_t$ = width of tooth on pitch circle (in.)

$\theta_p$ = angular backlash of pinion (rad)

$\theta_g$ = angular backlash of gear (rad)

$r_p$ = pitch radius of pinion (in.)

$r_g$ = pitch radius of gear (in.)

$\Delta C$ = increase in center distance above nominal (in.)

$\Phi_o$ = operating pressure angle (deg) $\approx \Phi_s$

$\Phi_s$ = standard pressure angle (deg)($14.5^\circ$, $20^\circ$, etc.)

C = nominal center distance (in.)(sum of pitch radius of gear and pinion)

$B_g$ = linear backlash per gear (in.)

$e_t$ = total composite error (in.)

$\theta_1$ = angular backlash of gear 1 (rad)

$\theta_{1:2}$ = angular backlash of gear 1 transferred to gear 2 (rad)

$N_2$ = velocity of gear 2 (rpm)

$N_1$ = velocity of gear 1 (rpm)

m (subscript) = maximum

p (subscript) = probable or pinion

## SUMMARY

The two principal problems which occur in the design of a mechanical drive for a servomechanism are the reduction of the inertia of the system and the reduction of the backlash of its gear train to a practical minimum. Information that will aid in the solution of these problems is included in this thesis.

A study of all available literature of inertia and backlash was made; the information that was obtained was consolidated into this paper. The effects, sources, methods of calculation, and methods of reduction of inertia and backlash in a servo system are included.

Inertia in a servo system reduces its speed of response, produces dynamic positional errors, and causes oscillations in the system. Excessive gear backlash produces system instability, oscillation, and static positional inaccuracy.

The inertia of a system can be reduced by the proper selection of gear-train ratios, gear materials, gear shape and size, and low inertia electrical components. Backlash can be reduced by the use of precision gears and ball bearings, close machining tolerances, proper materials, and antibacklash gears.

# CHAPTER I

## INTRODUCTION

In the design of an instrument servomechanism, it is necessary to reduce the inertia of the system and the backlash of its gear train to a practical minimum in order to obtain an accurate and stable system. Inertia in a servo system reduces its speed of response, produces dynamic positional errors, and causes oscillations in the system. Excessive gear backlash produces system instability, oscillation, and static positional inaccuracy (2)*. This thesis was written for the purpose of aiding in the design of the mechanical drive of a servo system with minimum backlash and inertia.

Since there was no single source of information available on the minimization of inertia and backlash, a study was made of all available books and periodicals that contained information on backlash and inertia; the facts and data that were obtained from this literature were consolidated and organized into this paper.

Sources, methods of calculation, and methods of reduction of inertia and backlash are included in the following chapters.

The inertia of a system can be reduced by the proper selection of gear-train ratios, gear materials, gear shape and size, and low inertia

---

*Numbers in parenthesis identify references listed in Bibliography.

electrical components. Backlash can be reduced by the use of precision gears and ball bearings, close machining tolerances, proper materials, and antibacklash gears.

# CHAPTER II

# INERTIA

## Definition and Effects

Inertia is the property of a body which opposes any change in the body's motion (1).

In order to obtain a maximum speed of response and to prevent oscillation in a servo system, it is necessary to reduce the inertia of the system to a minimum. A high speed of response requires a rapid system acceleration. To obtain the maximum acceleration of an inertia load with a motor torque, the ratio of torque-to-inertia at the load shaft or at the motor shaft must be a maximum, since $\alpha = T/I$. When a gear train separates the motor and the load, it must be designed with the proper ratio and with minimum overall inertia to obtain maximum system acceleration (4).

## Calculation

Formulas and Rules.--During the design of a servo system, it is necessary to obtain the inertia of all moving components. The inertia of components such as motors, tachometer generators, potentiometers, resolvers, synchros, and ball bearings can be obtained from their manufacturer's literature; the inertia of components such as gears, shafts, and pulleys must be calculated.

The formulas given below are to be used to calculate the moment of inertia of some of the more common solid bodies required in servo dynamics (5):

a. Solid circular cylinder about its axis of symmetry (Fig. 1a):

$$I_{AA} = \frac{\pi d t D_o^4}{32} \tag{1}$$

b. Hollow circular cylinder about its axis of symmetry (Fig. 1b):

$$I_{AA} = \frac{\pi d t \ (D_o^4 - D_1^4)}{32} \tag{2}$$

c. Solid circular cylinder about an axis parallel to, and $X_o$ distance from, its axis of symmetry (Fig. 1c):

$$I_{BB} = \frac{\pi d t D_o^2 \ (D_o^2 + 8X_o^2)}{32} \tag{3}$$

d. Body of weight W having linear motion and attached to a cable wound around a drum or a pulley of pitch diameter $D_p$; or a rack of weight W meshing with a gear of pitch diameter $D_p$; inertia referred to axis of symmetry of drum, pulley, or gear (Fig. 1d):

$$I_{BB} = W \ (D_p/2)^2 \tag{4}$$

e. Inertia in weight units transformed into inertia in mass units:

$$I \ (\text{oz-in.-sec}^2) = \frac{I \ (\text{oz-in.}^2)}{386.04 \ \text{in./sec}^2} \tag{5}$$

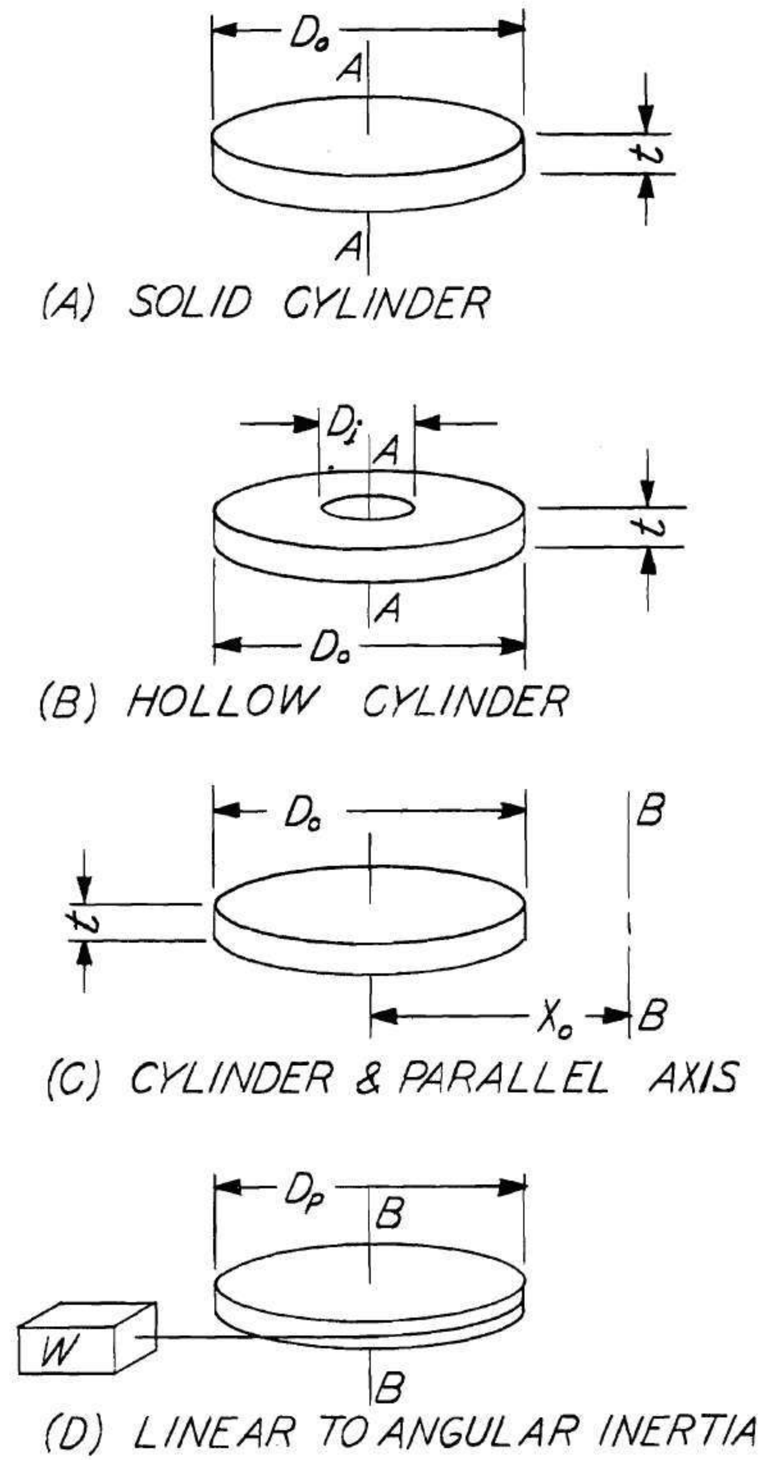


FIG. 1: INERTIA FIGURES

f. Symbols for above formulas:

$D_o$ = outside diameter of cylinder (in.)
$D_i$ = inside diameter of cylinder (in.)
t = thickness of cylinder (in.)
d = density of material of cylinder ($oz/in.^3$), (Table 1)
$X_o$ = perpendicular distance from axis of symmetry of the cylinder to a parallel axis (in.)
$D_p$ = pitch diameter of pulley, drum, or gear (in.)
W = weight of body or rack (oz)
$I_{AA}$ = moment of inertia of a body referred to its axis of symmetry ($oz\text{-}in.^2$)
$I_{BB}$ = moment of inertia of a body referred to an axis parallel to its axis of symmetry ($oz\text{-}in.^2$)

Fine pitch instrument gears usually are 48 diametral pitch or finer, have a $14.5^\circ$ or $20^\circ$ pressure angle, and have a face width of approximately one-eighth inch.

To calculate the inertia of a fine pitch gear, one should consider the gear a solid (or hollow) circular cylinder with an outside diameter equal to the pitch diameter of the gear and with a thickness equal to the face width of the gear (4).

$$D = \frac{n}{P} \tag{6}$$

D = pitch diameter of gear
n = number of teeth on gear
P = diametral pitch of gear

In the design of a servo gear train, it is often necessary to transfer inertias and torques from one shaft to another shaft of the gear train;

Table 1. Density of Materials

| Material | Density (d), (oz/in.$^3$) |
|---|---|
| Aluminum (24S-T3) | 1.600 |
| Brass (free cutting, yellow; Cu: .615; Zn: .355; Pb: .03) | 4.912 |
| Steel (stainless: 302, 303, 304) | 4.594 |
| Magnesium | 1.005 |
| Cast Iron (gray) | 4.160 |
| Copper | 5.184 |
| Lead | 6.555 |
| Nylon (Zytel 101, formerly FM10001) | 0.659 |
| Fibre (laminated thermosetting plastic with paper or cotton fabric base) | 0.751 - 0.786 |
| Bronze (Cu: .90; Zn: .10) | 5.088 |
| Water | 0.578024 |

it is also necessary to obtain a relationship between speeds and accelerations of various shafts. An example of a typical servo drive consisting of a motor, a load, and a two-mesh gear train is shown in Figure 2.

Usually the torque, acceleration, speed, and inertia of the load, and the inertia of the gear train are referred to the motor shaft. When transferred from a high speed shaft to a low speed shaft, inertia is decreased by the square of the gear ratio, torque is decreased by the gear ratio, and acceleration and speed are increased by the gear ratio. The reverse of the above is true when the transformation is made from a low speed shaft to a high speed shaft.

Symbols and Equations for Figure 2 (4):

$n_1, n_2, n_3, n_4$ = number of teeth on gear 1, 2, 3, 4, respectively

$d_1, d_2, d_3, d_4$ = density of gear 1, 2, 3, 4, respectively (oz/in.$^3$)

$D_1, D_2, D_3, D_4$ = pitch diameter of gear 1, 2, 3, 4, respectively (in.)

$t_1, t_2, t_3, t_4$ = thickness of gear 1, 2, 3, 4, respectively (in.)

$R = R_A R_B$ = overall gear ratio between motor and load (7)

$R_A = \frac{n_2}{n_1} = \frac{D_2}{D_1}$, $R_B = \frac{n_4}{n_3} = \frac{D_4}{D_3}$ = gear ratio of mesh A and B, respectively (8, 9)

$T_M$ = motor output torque (in.-oz)

$T_L$ = load friction torque (in.-oz); (may be a constant value or may be proportional to the load velocity)

$T_{LM}$ = load torque referred to motor shaft (in.-oz)

$N_M$, $N_L$ = speed of motor and load, respectively, (rpm)

$I_M$, $I_L$ = inertia of motor and load, respectively (oz-in.-sec$^2$)

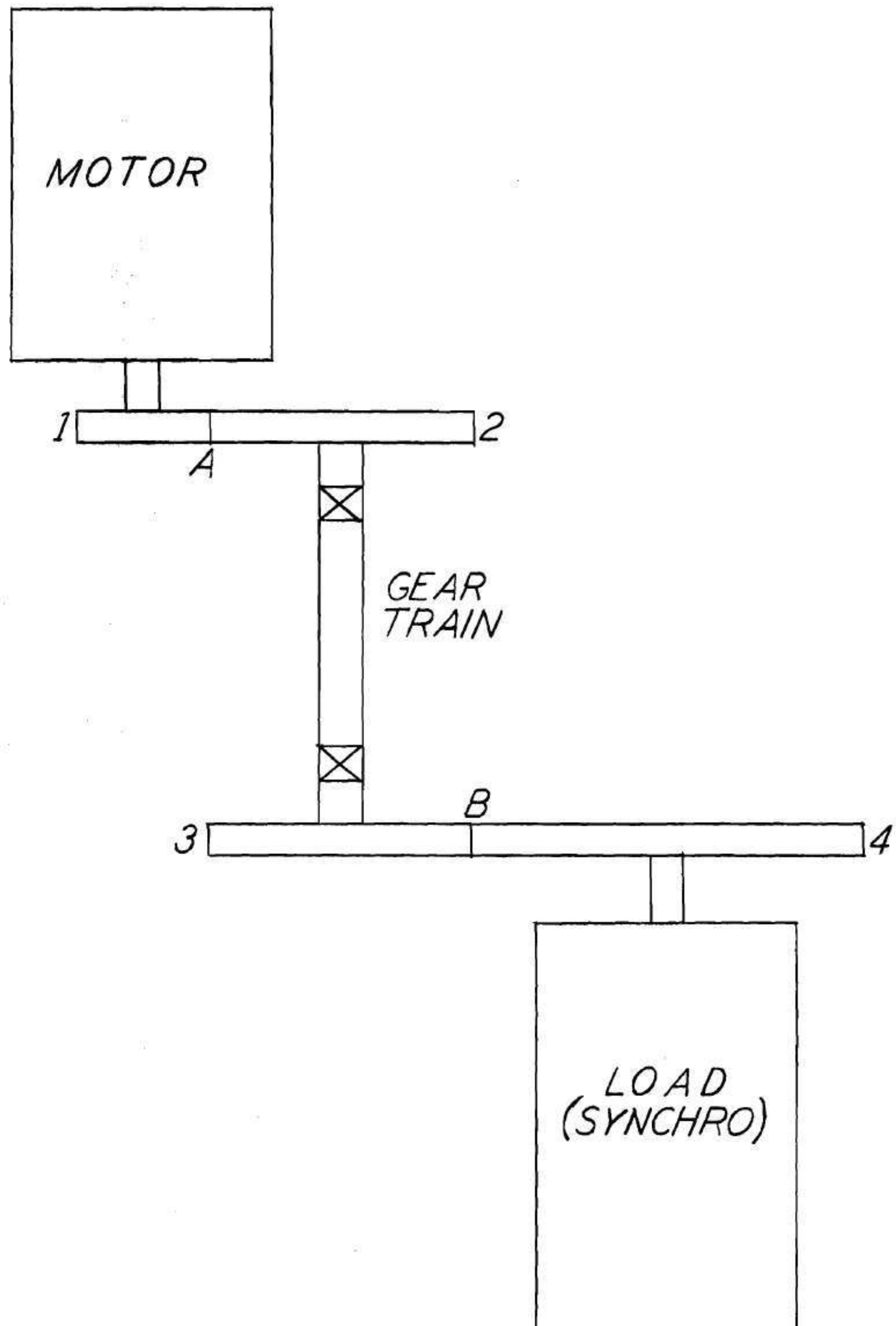


FIG. 2: SERVO DRIVE

$I_1$, $I_2$, $I_3$, $I_4$ = inertia of gear 1, 2, 3, 4, respectively (oz-in.-sec$^2$)

$I_{LM}$ = load inertia referred to motor shaft (oz-in.-sec$^2$)

$I_{2M}$, $I_{3M}$, $I_{4M}$ = inertia of gear 2, 3, 4, respectively, referred to motor shaft (oz-in.-sec$^2$)

$I_{23}$ = inertia of jack shaft between gear 2 and gear 3, (oz-in.-sec$^2$)

$I_{23M}$ = inertia $I_{23}$ referred to motor shaft (oz-in.-sec$^2$)

$\alpha_L$ = load acceleration, (rad/sec$^2$)

$\alpha_M$ = motor acceleration, (rad/sec$^2$)

$$T_{LM} = \frac{T_L}{R} \quad \text{(neglecting gear inefficiencies)} \tag{10}$$

$$T_{LM} = \frac{T_L}{eR} \quad \text{(e = gear efficiency = approximately .95 per spur gear mesh)} \tag{11}$$

$$I_{LM} = \frac{I_L}{R^2} = \frac{I_L}{R_A^2 R_B^2} \tag{12}$$

$$I_{2M} = \frac{I_2}{R_A^2} \tag{13}$$

$$I_{23M} = \frac{I_{23}}{R_A^2} \tag{14}$$

$$I_{3M} = \frac{I_3}{R_A^2} \tag{15}$$

$$I_{4M} = \frac{I_4}{R^2} = \frac{I_4}{R_A^2 R_B^2} \tag{16}$$

$$N_M = RN_L \tag{17}$$

$$\alpha_M = R\alpha_L \tag{18}$$

General Equation of Motion.--When Newton's Second Law of Motion, $\Sigma T = I\alpha$, is applied to the motor and the above quantities referred to the motor shaft and gear efficiencies are neglected, an equation is obtained that represents the dynamics of the servo drive:

$$T_M - T_{LM} = (I_M + I_1 + I_{2M} + I_{23M} + I_{3M} + I_{4M} + I_{LM})\,\alpha_M \tag{19}$$

or

$$T_M - \frac{T_L}{R} = \left(I_M + I_1 + \frac{I_2}{R_A^2} + \frac{I_{23}}{R_A^2} + \frac{I_3}{R_A^2} + \frac{I_4}{R^2} + \frac{I_L}{R^2}\right)\quad (R\alpha_L) \tag{20}$$

$$I_1 = \frac{\pi d_1 t_1 D_1^4}{32} \tag{21}$$

$$I_2 = \frac{\pi d_2 t_2 D_2^4}{32} \tag{22}$$

$$I_3 = \frac{\pi d_3 t_3 D_3^4}{32} \tag{23}$$

$$I_4 = \frac{\pi d_4 t_4 D_4^4}{32} \tag{24}$$

$$K_A = \frac{d_2 t_2}{d_1 t_1}\,, \quad R_A = \frac{D_2}{D_1} \tag{25, 26}$$

$$\frac{I_2}{I_1} = \frac{d_2 t_2 D_2^4}{d_1 t_1 D_1^4} = K_A R_A^4 \tag{27}$$

$$K_B = \frac{d_4 t_4}{d_3 t_3}\,, \quad R_B = \frac{D_4}{D_3} = \frac{R}{R_A} \tag{28, 29}$$

$$\frac{I_4}{I_3} = \frac{d_4 t_4 D_4^4}{d_3 t_3 D_3} = K_B R_B^4 = K_B \frac{R^4}{R_A^4} \tag{30}$$

$$T_M - \frac{T_L}{R} = \left(I_M + I_1 + \frac{K_A R_A^4 I_1}{R_A^2} + \frac{I_{23}}{R_A^2} + \frac{I_3}{R_A^2} + \frac{K_B R^4 I_3}{R^2 R_A^4} + \frac{I_L}{R^2}\right) \quad (R\alpha_L) \tag{31}$$

$$T_M - \frac{T_L}{R} = \left(I_M + I_1 + K_A R_A^2 I_1 + \frac{I_{23}}{R_A^2} + \frac{I_3}{R_A^2} + \frac{K_B R^2 I_3}{R_A^4} + \frac{I_L}{R^2}\right) \quad (R\alpha_L) \tag{32}$$

$\alpha_L$ from Equation 32:

$$\alpha_L = \frac{\left(T_M - \frac{T_L}{R_A R_B}\right)}{R_A R_B \left(I_M + I_1 + K_A R_A^2 I_1 + \frac{I_{23}}{R_A^2} + \frac{I_3}{R_A^2} + \frac{K_B R_A^2 R_B^2 I_3}{R_A^4} + \frac{I_L}{R_A^2 R_B^2}\right)} \tag{33}$$

## Design for Minimum Inertia

Exact Method.--Equation 33 is the general equation of acceleration of the servo drive shown in Figure 2. The load torque $T_L$ is usually small and can be neglected in the initial calculations. The gear ratios $R_A$ and $R_B$ for maximum load acceleration can be found by the simultaneous solution of the equations (4):

$$\frac{\partial \alpha_L}{\partial R_A} = 0, \quad \frac{\partial \alpha_L}{\partial R_B} = 0 \tag{34}$$

The overall gear ratio R is then equal to $R_A R_B$.

When the required overall gear ratio R is initially known, it can be substituted into Equation 33 for one of the unknowns, $R_A$ or $R_B$, (See Equation 7); then from the solution of the equation,

$$\frac{\partial \alpha_L}{\partial R_A} = 0 \quad \text{or} \quad \frac{\partial \alpha_L}{\partial R_B} = 0 \quad , \tag{35}$$

one of the mesh ratios can be obtained, and the other mesh ratio can be found from Equation 7.

The above example included a gear train of only two meshes; a similar equation for a gear train of n meshes can be written and solved for the load acceleration $\alpha_L$. The partial derivative of $\alpha_L$ with respect to each of the gear-mesh ratios can be found and set equal to zero. The simultaneous solution of these partial derivative equations will give the individual gear mesh ratios for the n meshes to produce maximum load acceleration.

Approximate Method.--Sometimes it is desirable to neglect gear train inertia initially and to obtain a rapid approximation of the optimum gear ratio required to meet specific servo load specifications. Equations for calculating the approximate overall gear ratio required to meet given load specifications are given below (2):

a. For maximum steady state load velocity:

$$R = \frac{T_L}{T_{MF}} \tag{36}$$

b. For maximum acceleration of an inertia load:

$$R = (I_L/I_M)^{1/2} \tag{37}$$

c. For a specified maximum load velocity, $\omega_L$, and a specified maximum load acceleration, $\alpha_L$, with minimum motor torque, and with load torque and motor friction torque proportional to system velocity:

$$R = \frac{-T_{MF} + \left(T_{MF}^2 + 4\ (T_L + I_L\alpha_L)\ (I_M\alpha_L)\right)^{1/2}}{2(I_M\alpha_L)} \tag{38}$$

d. Symbols for above equations:

$R$ = gear ratio = $\frac{\text{motor speed}}{\text{load speed}}$

$I_L$ = load inertia (oz-in.-sec$^2$)

$I_M$ = motor inertia (oz-in.-sec$^2$)

$\alpha_L$ = load acceleration (rad/sec$^2$)

$T_L$ = load friction torque (in.-oz)

$T_{MF}$ = motor friction torque (in.-oz)

Meshes and Ratios.--The overall gear ratio between the motor and the load may be found by the above approximation, or it may be given in the specifications of the system. The number of gear meshes and the required ratio per mesh can be found from Figure 3 or Figure 4. Figure 3 and Figure 4 are constructed for the following gear train conditions: all pinions are the same size and are as small as practical without introducing excessive angular backlash; pinions and gears are made of the same density material and have the same face width. If these conditions are not met in a design problem, the figures may still be used to give an approximation of the gear mesh ratios; however, a new set of curves can be constructed which will satisfy the design specifications and will give exact gear-mesh ratios. (For details, see Reference No. 2 of Bibliography.)

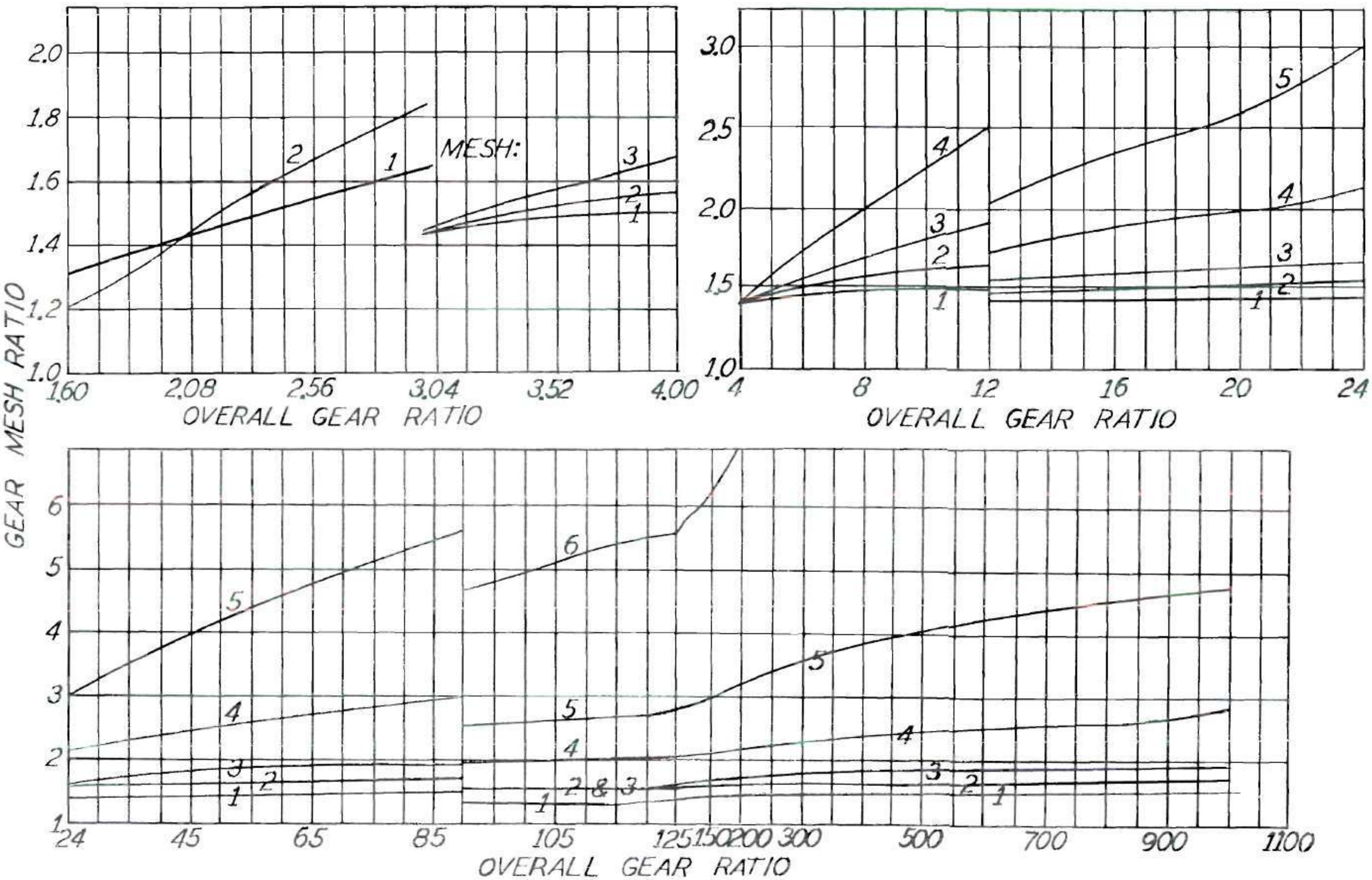


FIG.3: GEAR-MESH RATIOS

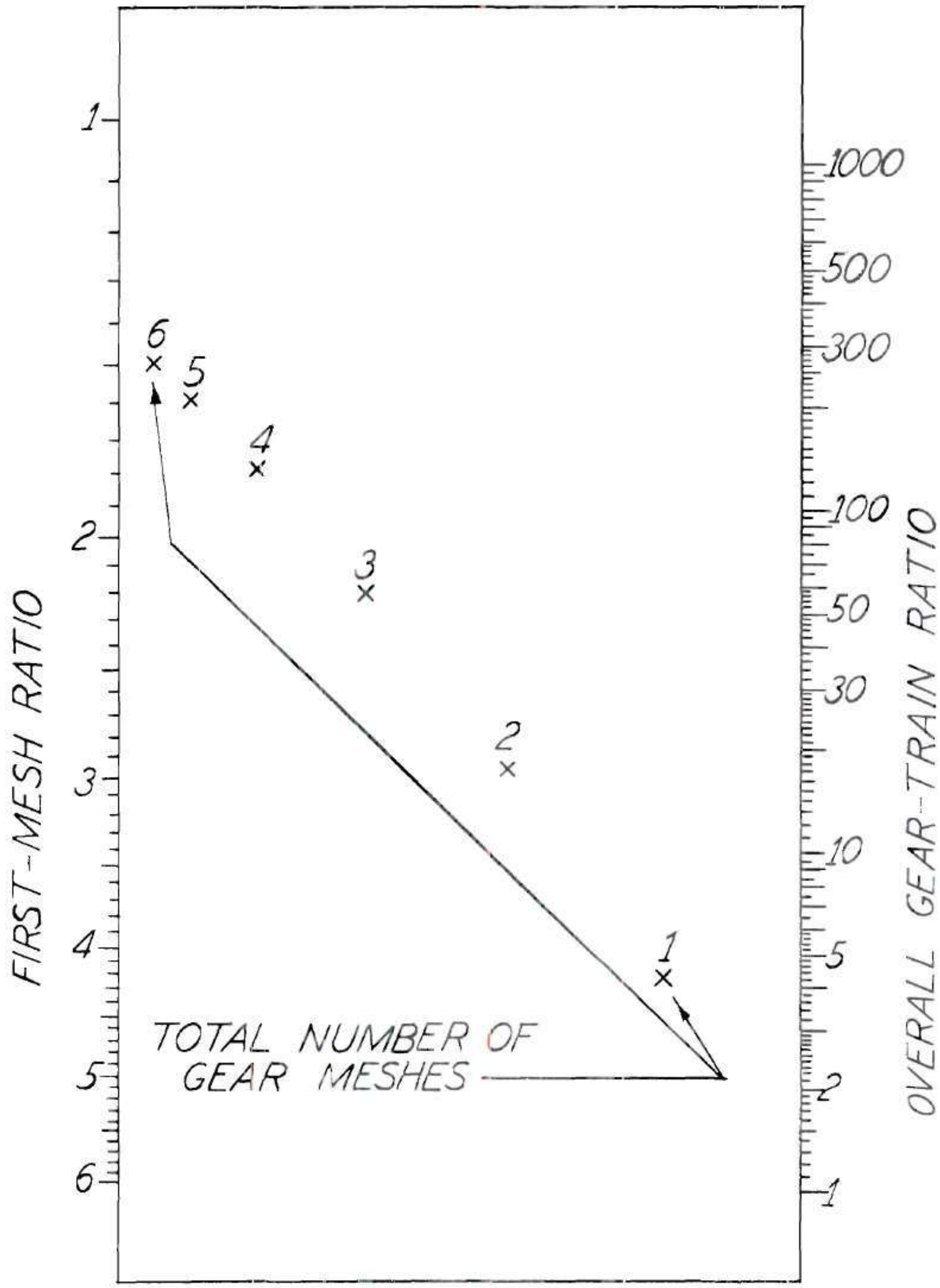


FIG. 4: GEAR-MESH-RATIO NOMOGRAM

If an absolute minimum value of gear-train inertia is desired, Figure 3 should be used to obtain the number of meshes and the ratio per mesh (2).

The overall gear ratio is located along the abscissa; a line drawn through this gear ratio value, parallel to the ordinate, intersects each of the mesh curves. Lines drawn through each of the points of intersection, parallel to the abscissa, intersect the ordinate at the gear-mesh ratio value for each of the meshes. Example: If the overall gear ratio equals 10, first mesh ratio (between motor and first jack shaft) is 1.5, second mesh ratio is 1.6, third mesh ratio is 1.8, fourth mesh ratio is 2.3.

If it is desirable, for economy, to use fewer than the optimum number of gear meshes, Figure 4 should be used (2). A line, drawn from the overall gear ratio (located on the right ordinate) through the total number of meshes (located between the ordinates), will intersect the left ordinate and indicate the ratio of the first mesh. The overall ratio is then divided by the first mesh ratio to obtain the remaining ratio. Then a line, drawn through the remaining ratio (located on the right ordinate) and through the number of remaining meshes (located between the ordinates), will intersect the left ordinate at the ratio of the second mesh. This procedure is repeated until the ratio of each mesh has been determined. Example: If the overall ratio equals 10 and three meshes are to be used, first mesh ratio is 1.72, second mesh ratio is 2.07, third mesh ratio is 2.81. This gear train will have the minimum inertia possible for an overall gear ratio of 10 with three meshes.

## Methods of Reduction

Materials.--Materials used for the manufacture of instrument gears should have minimum density consistent with the requirements for satisfactory durability and wear. Many instrument gears are now being made of aluminum and nylon, and their pinions are being made of stainless steel. The densities of various materials are given in Table 1.

Some of the properties of nylon that make it an ideal material for instrument gears are listed below (8):

1. Low density and therefore low inertia
2. No lubrication required
3. Noise and vibration damper
4. Low coefficient of friction (nylon and nylon, dry: 0.04 - 0.10; nylon and steel and SAE 10 oil: 0.14)
5. Corrosion resistant and good wearability
6. Molded parts tougher than machined parts

Gear Shape.--The inertia of a gear may be reduced by removing material from the web between the gear teeth and the gear hub. The web thickness may be reduced as shown in Figure 5 (here the inertia of the gear was reduced to .65 of its initial value); holes may be drilled through the web as shown in Figure 6 (here the inertia was reduced to .75 of its initial value).

If too much material is removed or if it is removed from too near the gear teeth, the strength and the rigidity of the gear will be overly reduced, and axial and radial runout of the gear may result. To avoid the

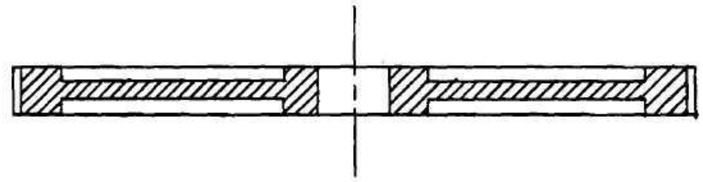

FIG.5: LOW-INERTIA GEAR

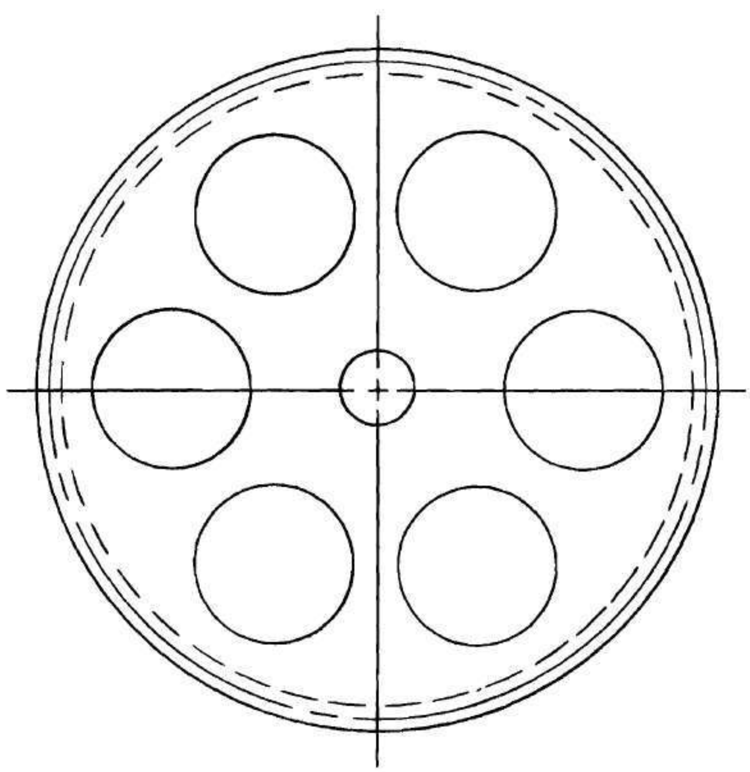

FIG. 6: LOW-INERTIA GEAR

possibility of gear distortion, it is advisable to reduce the weight of the web of a gear blank before the final outside diameter of the gear is turned, and thus before the gear teeth are cut.

Gear Ratio--It has been stated previosly that the inertia of a servo system can be reduced to a minimum by the proper selection of the overall gear ratio and of the gear-mesh ratios. Approximate values of the proper overall and mesh ratios can be found as stated above; to obtain exact overall and mesh ratios for minimum system inertia often requires a trial-and-error solution, using the approximate values as initial values in the solution. The approximate values are often satisfactory for almost all servo system designs.

# CHAPTER III

# BACKLASH

## Effects and Definition

The second of the problems encountered in the design of a servomechanism precision gear train is the minimization of gear backlash. Excessive backlash produces instability, oscillation, and positional inaccuracy in the servo system. In many electromechanical servo control circuits, the gearing is the limit to the overall accuracy and performance of the system.

Linear backlash is defined as the shortest distance between the non-driving tooth surfaces of adjacent teeth on meshing gears or the amount by which the width of a tooth space exceeds the thickness of the engaging tooth, measured in inches on the pitch circle. Angular backlash is measured in radians and is equal to the linear backlash divided by the pitch radius of either gear and therefore must be referred to a specific gear of the meshed pair. Angular backlash can be defined as the angle through which a meshed gear can be rotated when its mating gear is held fixed (18).

$$B = T_s - T_t \tag{39}$$

$$\theta_p = \frac{B}{r_p} \tag{40}$$

$$\theta_g = \frac{B}{r_g} \tag{41}$$

$B$ = linear backlash of meshed pair (in.)

$T_s$ = width of tooth space on pitch circle (in.)

$T_t$ = width of tooth on pitch circle (in.)

$\theta_p$ = angular backlash of pinion (rad)

$\theta_g$ = angular backlash of gear (rad)

$r_p$ = pitch radius of pinion (in.)

$r_g$ = pitch radius of gear (in.)

A theoretically perfect gear, mounted perfectly, would require no backlash to operate properly, and its tooth thickness, measured on the pitch circle, would be equal to one-half the circular pitch of the gear. However, because of unavoidable manufacturing inaccuracies, it is necessary to provide some clearance (backlash) so that the gears will not bind but will roll together smoothly. Backlash is intentionally secured by setting the cutting tool somewhat deeper into the gear blank than the theoretical position, thus cutting the teeth slightly thinner; backlash may also be secured by having the center distance of the meshing gears slightly oversize. This backlash is necessary and desirable, but an additional amount of backlash, which is both unintentional and undesirable, is often introduced into gears. This undesirable backlash should be held to a minimum during the manufacturing operations, and it may be eliminated with antibacklash methods upon assembly.

## Sources of Backlash

The sources of backlash in gear trains are listed below:

1. Gear center-distance variation

2. Gear tooth size
3. Pitch diameter runout
4. Ball bearing errors
5. Gear assembly to shaft
6. Shaft runout
7. Looseness of shaft, bearings, and housing bore
8. Composite gears
9. Environmental sources
10. Rigidity of installation

<u>Gear Center-Distance Variation</u>.--This source is usually the largest of the above backlash components. Regardless of the machining method used, it is impossible to obtain the exact nominal theoretical center distance in practice; therefore, to avoid possible gear interference, the tolerance on gear center distance is usually given as the theoretical nominal (equal to the sum of the pitch radii of the two meshing gears) to plus some specified allowance of 0.0003 inch to 0.001 inch, depending on the type of machining operation. The center distance will then be just nominal or greater than nominal because of the manufacturing tolerances.

The following tolerances can be obtained on center distances up to five inches:

1. Jigs and fixtures ................ ± 0.001 - inch
2. Jig boring ....................... ± 0.0003 - inch
3. Special production techniques ..... ± 0.0005 - inch

In order to avoid interferences which could result from tolerances or from pitch circle runout, the center distance is sometimes intentionally increased beyond its nominal value.

Any increase in gear center distance above nominal, either due to manufacturing tolerances or to an intentional increase for smoother operation, produces gear backlash. The backlash is given by the following formula (19):

$$B = 2(\Delta C) \tan \Phi_o \tag{42}$$

$B$ = linear backlash measured on the pitch circle, (in.)

$\Delta C$ = increase in center distance above nominal, (in.)

$\Phi_o$ = operating pressure angle, (deg) $\approx \Phi_s$

$\Phi_s$ = standard pressure angle (deg) ($14.5^\circ$, $20^\circ$, etc.)

An involute gear has no pressure angle or pitch diameter until it is meshed with another involute gear at a fixed center distance. If this center distance is greater than the nominal center distance, then the operating pressure angle ($\Phi_o$) of the gears will exceed the standard pressure angle ($\Phi_s$) of the tooth generating tool; thus changing the center distance changes the operating pressure angle which alters the backlash. The operating pressure angle can be calculated with the following formula (19):

$$\text{COS } \Phi_o = \frac{\text{COS } \Phi_s \ (C)}{(C + \Delta C)} \tag{43}$$

$\Phi_o$ = operating pressure angle $\approx \Phi_s$

$\Phi_s$ = standard pressure angle ($14\text{-}1/2^\circ$, $20^\circ$, etc.)

$C$ = nominal center distance (sum of pitch radius of gear and pinion)

$\Delta C$ = increase in center distance above nominal

Gear Tooth Size.--Variations in the size of gear teeth are also one of the major sources of backlash. Theoretically, perfectly made gears, operating on exact nominal center distances, would require no backlash; and the thickness of their teeth would equal one-half their circular pitch ($t = \pi/2P$). However, actual teeth are usually specified slightly thinner to provide an intentional design backlash and thus avoid the possibility of interference; a manufacturing tolerance is also given which permits the teeth to be even thinner and thus increases the possible backlash.

Backlash which is due to a reduction in the tooth size of a gear can be measured by mounting the gear on a variable-center-distance inspection fixture. The gear, on a spring-loaded floating shaft which meshes the gears, is rolled in mesh with a master gear; the difference, if any, between their nominal center distance and their actual center distance is measured, and the backlash is calculated by using Equations 42 and 43.

The standard specified backlash for various classes of fine pitch gears is given in Table 2 and Table 3; these values are from the American Gear Manufacturers' Association Standard 236.03.

These tables indicate that backlash will approach a minimum when a precision class of fine pitch gears is used; the finer the pitch and the more precise the class, the less backlash will be produced by a reduction in gear tooth thickness.

Table 2. Standard Specified Backlash

| Diametral Pitch | Backlash* (inches) |
|---|---|
| Class A | |
| 20 - 40 | 0.004 - 0.006 |
| 41 - 70 | 0.003 - 0.005 |
| 71 - 90 | 0.002 - 0.0035 |
| Class B | |
| 20 - 60 | 0.002 - 0.004 |
| 61 - 120 | 0.0015 - 0.003 |
| 121 - finer | 0.001 - 0.002 |
| Class C | |
| 20 - 60 | 0.001 - 0.002 |
| 61 - 120 | 0.0007 - 0.0015 |
| 121 - finer | 0.0005 - 0.001 |
| Class D | |
| any pitch | no measurable backlash |

*Between two assembled gears at their tightest point of mesh. Backlash will be increased when the low points of runout are in contact.

Table 3. Backlash Components Converted from Table 2

| Diametral Pitch | Intentional Backlash (in. per pair) | Backlash Tolerance (in. per pair) | Probability Factor for Tolerance |
|---|---|---|---|
| Class A | | | |
| 20 - 45 | 0.004 | 0.002 | 0.5 |
| 46 - 70 | 0.003 | 0.002 | 0.5 |
| 71 - 90 | 0.002 | 0.0015 | 0.7 |
| Class B | | | |
| 20 - 60 | 0.002 | 0.002 | 0.5 |
| 61 - 120 | 0.0015 | 0.0015 | 0.6 |
| 121 - finer | 0.001 | 0.001 | 0.8 |
| Class C | | | |
| 20 - 60 | 0.001 | 0.001 | 0.8 |
| 61 - 120 | 0.0007 | 0.0008 | 1.0 |
| 121 - finer | 0.0005 | 0.0005 | 1.0 |

Pitch Diameter Runout.--It is almost impossible to produce a gear without having the pitch circle vary slightly and without having some eccentricity between the pitch circle and the bore. The pitch circle, the circumferential trace through the gear teeth designating constant tooth thickness, becomes slightly eccentric relative to the bore center and also departs from being perfectly circular due to tooth-to-tooth variations.

Any eccentricity will cause the gear teeth to work slightly in and out of mesh and the backlash to vary as the gear rotates; the backlash will be increased as the short radius part of the gear passes through the mesh point where the thinner outer portions of the gear teeth are in contact. Additional backlash will be caused by any tooth thickness variation, profile error, tooth spacing variation, or lateral gear runout.

The maximum overall variation of the pitch radius of a gear (maximum pitch radius minus minimum pitch radius) is called the total composite error; Table 4 shows limiting values of total composite error from AGMA Standard 236.03. The pitch radius variation of a gear can be measured by measuring the change in center distance as the gear is rotated in mesh with a master gear on a variable-center-distance inspection fixture. The backlash contribution at the low point of each of a pair of meshed gears due to total composite error is given by the formula below (18):

$$B_g = e_t \tan \Phi_o \tag{44}$$

$B_g$ = linear backlash per gear

$e_t$ = total composite error

$\Phi_o$ = operating pressure angle $\approx \Phi_s$

Table 4. Total and Tooth-to-Tooth Composite Error Limits

| Class | Total Composite Error (in.) | Tooth-to-Tooth Composite Error (in.) | Probability Factor for Total Composite Error |
|---|---|---|---|
| Commercial 1 | 0.006 | 0.002 | 0.5 |
| Commercial 2 | 0.004 | 0.0015 | 0.5 |
| Commercial 3 | 0.002 | 0.001 | 0.67 |
| Commercial 4 | 0.0015 | 0.0007 | 0.67 |
| Precision 1 | 0.001 | 0.0004 | 0.67 |
| Precision 2 | 0.0005 | 0.0003 | 0.8 |
| Precision 3 | 0.00025 | 0.0002 | 1.0 |

Table 5. Ball Bearing Runout*

| ABEC Class | Inner Race Eccentricity (in. TIR) | Outer Race Eccentricity (in. TIR) | Probability Factor |
|---|---|---|---|
| 1 | 0.0003 | 0.0006 | 1 |
| 3 | 0.0002 | 0.0004 | 1 |
| 5 | 0.0002 | 0.0002 | 1 |
| 7 | 0.0001 | 0.0002 | 1 |

*For bores to 1/4 inch only

Ball Bearing Errors.--Eccentricity of the inner and/or outer race of a ball bearing affects the backlash of a pair of meshed gears. If the outer race is fixed and the inner race rotates, any runout of the inner race will cause the center of a gear mounted to a shaft in the bearing to have a small circular motion as the gear rotates; this motion gives the pitch circle an additional amount of eccentricity and produces backlash which varies with gear rotation.

Any runout of the outer race will produce a permanent shift of the gear center which will alter the center distance between the gear and its meshing gear. The magnitude and direction of the center shift will depend on the angular orientation of the outer race with respect to the mesh point of the gears; the center shift produces backlash in the same manner as gear center-distance variation.

Radial play in ball bearings can also add to gear backlash. Table 5 gives limiting values of ball bearing runout. Since the runout and radial play of small precision ball bearings are usually very small, ball bearings have little effect on the total backlash of a gear mesh.

Gear Assembly to Shaft.--If any clearance exists between a gear bore and its mounting-shaft diameter, it is possible that the assembly will be eccentric when assembled, regardless of the method of fastening. The result will be runout of the pitch circle of the gear, which will produce backlash that will vary with gear rotation. This backlash source can be eliminated by reducing the clearance to zero and specifying a press fit of the shaft and gear bore. Table 6 shows shaft and bore di-

Table 6. Clearance Between Shaft and Gear

| AGMA Class | Nominal Shaft Dia. (in.)* | Nominal Gear Bore (in.)* | Maximum Clearance (in.) | Maximum Gear Center Shift (in.) | Probability Factor |
|---|---|---|---|---|---|
| Commercial 1 | + 0.0001<br>- 0.0002 | + 0.0021<br>+ 0.0001 | 0.0023 | ± 0.0012 | 0.50 |
| Commercial 2 | + 0.0001<br>- 0.0002 | + 0.0011<br>+ 0.0001 | 0.0013 | ± 0.0006 | 0.50 |
| Commercial 3 | + 0.0001<br>- 0.0002 | + 0.0008<br>+ 0.0001 | 0.0010 | ± 0.0005 | 0.67 |
| Commercial 4 | + 0.0001<br>- 0.0002 | + 0.0008<br>+ 0.0001 | 0.0010 | ± 0.0005 | 0.67 |
| Precision 1 | + 0.0000<br>- 0.0002 | + 0.0006<br>+ 0.0001 | 0.0008 | ± 0.0004 | 0.67 |
| Precision 2 | + 0.0000<br>- 0.0002 | + 0.0003<br>+ 0.0001 | 0.0005 | ± 0.00025 | 0.75 |
| Precision 3 | + 0.0000<br>- 0.0002 | + 0.0003<br>+ 0.0001 | 0.0005 | ± 0.00025 | 0.75 |

*The above shaft and bore dimensions are the values to be added to the nominal dimension to obtain the limiting dimensions of the shaft diameter and the gear bore.

mensions, tolerances, and maximum clearances for various AGMA gear classes. When precision gears and shafts are used, this source of backlash is small.

Shaft Runout.--Shafts, if long and slender and/or if stepped for bearing mounting, have a measurable amount of runout between their bearing mounting diameters and their gear mounting diameters. This runout will produce a cyclic variation of backlash with gear rotation; however, since the runout is usually small, its effect on backlash is also small. Typical shaft runout values for various components are shown in Table 7.

Looseness of Shaft, Bearing, and Housing Bore.--A loose fit, between the shaft of a gear and the inside diameter of a bearing or between the outside diameter of the bearing and the housing bore, can cause backlash. When a load is transmitted by the gears, their centers are forced apart; this increase in center distance produces backlash. For precision assemblies, this looseness is very small, and its contribution to gear backlash is likewise small.

Composite Gears.--An assembly consisting of a gear and a separate hub will contribute to the toal backlash because of the eccentricity of the assembly. If the assembly is a clearance fit, rather than a press fit, additional eccentricity and backlash will exist.

Environmental Sources.--If gear trains are required to operate throughout a wide range of ambient temperature, dimensional changes will result from the thermal coefficient of linear expansion. If the gears, shafts, and housings are not constructed of the same material, backlash may be affected because of differences in thermal expansion.

Table 7. Shaft Runout for Typical Components

| Component | Maximum TIR at Shaft End, (in.) | Probability Factor |
|---|---|---|
| Precision synchros | 0.001 | 0.7 |
| Precision resolvers | 0.0002 | 1.0 |
| Precision potentiometers | 0.0005 | 0.8 |
| Stepped shafts | | |
| Lathe-turned shoulders | 0.001 | 0.8 |
| Precision-turned shoulders | 0.0005 | 1.0 |
| Precision-ground shoulders | 0.0002 | 1.0 |
| Precision centerless-ground shafting | 0.0001 (per inch of length) | 1.0 |

Rigidity of Installation.--The deflection of long slender shafts and unbraced bearing housings, which occurs when the gears transmit a load, is an additional source of backlash.

Classification and Sum of Backlash Sources.--Table 8 gives three classifications of the previously listed backlash sources. Group A sources are design sources and are determined by the gear designer. These sources produce known amounts of backlash, which can be added to obtain their total. Group B sources are tolerance sources and are due to variations in the machining operations. The backlash produced by these sources can be added directly after applying the probability factors listed in Table 3 through Table 7. Group C sources are also tolerance sources, but they produce backlash that is not constant in magnitude but varies with gear rotation. The following procedure will give their contribution to the total backlash (18):

a. Apply probability factors to each source of backlash.

b. Add these components and apply an assembly phasing-factor of 0.7.

c. After performing the above steps for each gear of a meshed pair, apply a meshing phasing factor of 0.7 to the sum of the probable errors obtained for each gear for velocity ratios under 2, and a factor of 1 for higher ratios.

d. Since the above steps give the total probable eccentricity (or change in the center distance of the gears), apply Equations 42 and 43 to obtain the backlash contribution due to this eccentricity.

The total net probable linear backlash of a meshed pair of gears is found by adding the contributions of the sources of Groups A, B, and C. Group C sources are sometimes neglected because of their small contribution

Table 8. Classification of Backlash Sources

Group A - Design Components

1. Any specific increase in center distance nominal
2. Reduction of tooth thickness from nominal

Group B - Tolerance Components

1. Tolerance on tooth thickness
2. Tolerance on center distance
3. Bearing center shift due to outer race eccentricity
4. Radial clearance in bearing
5. Tolerances on shaft diameter, bearing diameters, and bearing housing bore diameter

Group C - Components Variable with Gear Rotation

1. Total composite error of pitch circle
2. Clearance fit between gear bore and shaft
3. Shaft runout at point of gear mounting
4. Bearing inner-race eccentricity
5. Eccentricity of composite gear assembly

to the total backlash. The total angular backlash in radians of a mesh is found by dividing the total linear backlash by the pitch radius of the gear in the mesh to which the angular backlash is referred (Equations 40 and 41). The total angular backlash of a gear train is usually found by taking the sum of the angular backlash of all meshes after the backlash has been transferred to the motor shaft. The following formula is used to transfer angular backlash from one gear to another gear in a gear train:

$$\theta_{1:2} = \theta_1 \frac{N_2}{N_1} \tag{46}$$

$\theta_1$ = angular backlash of gear 1, (rad)

$\theta_{1:2}$ = angular backlash of gear 1 transferred to gear 2, (rad)

$N_2$ = velocity of gear 2, (rpm)

$N_1$ = velocity of gear 1, (rpm)

The total angular backlash of a gear train referred to the motor shaft signifies the number of radians, or degrees, through which the motor shaft could be rotated while the load shaft is held stationary.

Calculation

Example Problem.--A typical servo gear train, shown in Figure 7, will be used to demonstrate the calculation of backlash. The probable and the maximum possible backlash of gear mesh A will be calculated; directions for the calculation of the backlash of mesh B and the total system back-

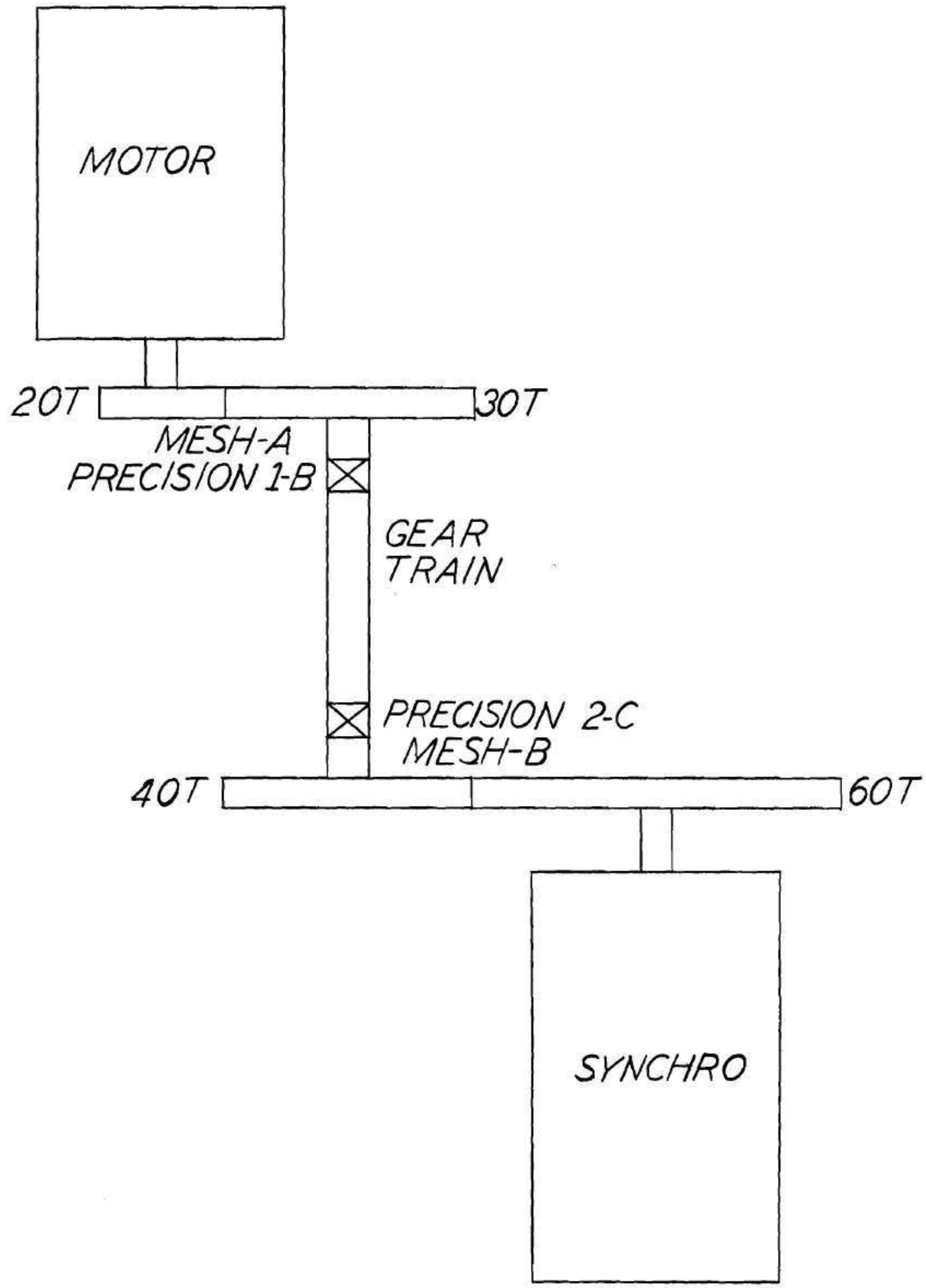


FIG.7: SERVO GEAR TRAIN

lash will be given also.

The following data, as well as that shown in Figure 7, describes the gear train.

1. The gears on the motor and the synchro are mounted on their shaft ends; both shaft ends run true within 0.001-inch TIR (Total Indicator Reading) and have a radial play of 0.0005-inch maximum.

2. The gears on the jack shaft are mounted one-half inch from the ball bearings; the ball bearings are ABEC-5 with 0.0003 to 0.0006-inch radial play; the jack shaft is a 4-inch long centerless ground shaft which is accurate to 0.0001 in./in. TIR.

3. All gears are 48 diametral pitch and 20-degree pressure angle; the number of teeth and the class of precision of each gear are shown in Figure 7; each is clamped to its shaft with the recommended fit shown in Table 6.

4. The gear-mesh center distances are nominal, with a tolerance of $\frac{-0.000}{+0.001}$ - inch.

5. Both the synchro and the motor mounting diameters are $2.0000\ \frac{+0.0000}{-0.0015}$ - inch, and they fit into housing bores of $2.003\ \frac{+0.0007}{-0.0000}$ - inch.

6. Total indicator reading (TIR), diametral clearance, and radial play values must be halved in order to obtain the radial eccentricity values.

7. A probability factor must be used to obtain the probable values of those backlash components which can vary in magnitude.

8. Phasing factors must be applied to the probable backlash values of Group C sources.

9. Variations in center distance must be converted into variations in backlash by the use of Equation 42 and Equation 43.

10. The words pinion and gear, at the right of the backlash values in the following calculations, indicate to which of the gears of the mesh the backlash refers.

Equations and Solution for Mesh A.--

1. Group A and Group B backlash sources:

$\Delta C$ = variation in center distance (in.)

B = linear backlash (in.)

m = maximum

p = probable

a. Center distance variation, (backlash due to intentional variation above nominal plus tolerance variation):

$$\Delta C_m = 0.0000 + 0.0010 = 0.0010$$

$$B_m = 2(\Delta C_m) \tan \Phi = 2(0.0010) \tan 20^\circ = 0.00073$$

$$\Delta C_p = 0.0000 + (0.0010)(0.5) = 0.0005$$

$B_p = 2(0.0005)(0.364) = 0.00037$ (pinion and gear)

b. Gear tooth size (intentional backlash plus tolerance backlash from Table 3 for 48 diametral pitch and Class B gears):

$B_m = 0.0020 + 0.0020 = 0.0040$ (pinion and gear)

$B_p = 0.0020 + 0.0020\ (0.5) = 0.0030$ (pinion and gear)

c. Clearance of motor in mounting hole (backlash due to intentional radial clearance plus tolerance radial clearance):

$$\Delta C_m = \frac{0.0003}{2} + \frac{0.0007 + 0.0015}{2} = 0.0013$$

$B_m = 2(0.0013)(0.364) = 0.00095$ (pinion)

$$\Delta C_p = \frac{0.0003}{2} + \frac{(0.0007 + 0.0015)(0.5)}{2} = 0.00070$$

$$B_p = 2(0.00070)(0.364) = 0.00051 \quad \text{(pinion)}$$

d. Motor shaft radial play (backlash due to radial eccentricity):

$$\Delta C_m = \frac{0.0005}{2} = 0.00025$$

$$B_m = 2(0.00025)(0.364) = 0.00018 \quad \text{(pinion)}$$

$$\Delta C_p = \frac{(0.0005)(0.5)}{2} = 0.00013$$

$$B_p = 2(0.00013)(0.364) = 0.00009 \quad \text{(pinion)}$$

e. Radial play of the ball bearing that supports the 30-tooth gear (backlash due to radial eccentricity):

$$\Delta C_m = \frac{0.0006}{2} = 0.0003$$

$$B_m = 2(0.0003)(0.364) = 0.00022 \quad \text{(gear)}$$

$$\Delta C_p = \frac{0.0003}{2} + \frac{(0.0003)(0.67)}{2} = 0.00025$$

$$B_p = 2(0.00025)(0.364) = 0.00018 \quad \text{(gear)}$$

f. Outer race eccentricity of the ball bearing that supports the 30-tooth gear (backlash due to bearing radial eccentricity, from Table 5 for a Class 5 bearing):

$$\Delta C_m = \frac{0.0002}{2} = 0.0001$$

$$B_m = 2(0.0001)(0.364) = 0.00007 \quad \text{(gear)}$$

$$\Delta C_p = \frac{(0.0002)(1.0)}{2} = 0.0001$$

$$B_p = 2(0.0001)(0.364) = 0.00007 \quad \text{(gear)}$$

g. Sum of backlash from Group A and Group B sources (sum of components a through f above):

$$\Sigma B_m = 0.00073 + 0.00400 + 0.00095 + 0.00018 + 0.00022 + 0.00007 = 0.00615$$

$$\Sigma B_p = 0.00037 + 0.00300 + 0.00051 + 0.00009 + 0.00018 + 0.00007 = 0.00422$$

2. Group C backlash sources:

a. Total composite error (sum of backlash due to total composite error of each gear, from Equation 44 and Table 4 for a Precision 1 gear):

$$e_{tm} = 0.0010/\text{gear}$$

$$B_m = (0.0010)(0.364) + (0.0010)(0.364) = 0.00072$$
(pinion and gear)

$$e_{tp} = (0.0010)(0.67) = 0.00067/\text{gear}$$

$$B_p = (0.00067)(0.364) + (0.00067)(0.364) = 0.00048$$
(pinion and gear)

b. Radial clearance between gear bore and shaft (sum of backlash due to gear center shift of each gear, Table 6 for Precision 1 gear):

$$\Delta C_m = 0.0004/\text{gear}$$

$$B_m = 2(0.0004)(0.364) + 2(0.0004)(0.364) = 0.00058$$
(pinion and gear)

$$\Delta C_p = (0.0004)(0.67) = 0.00027/\text{gear}$$

$$B_m = 2(0.00027)(0.364) + 2(0.00027)(0.364) = 0.00040$$
(pinion and gear)

c. Motor shaft runout (backlash due to radial eccentricity of the shaft; assume probability factor for synchro from Table 7 since factor for motor is not given):

$$\Delta C_m = \frac{0.0010}{2} = 0.0005$$

$$B_m = 2(0.0005)(0.364) = 0.00036 \quad \text{(pinion)}$$

$$\Delta C_p = \frac{(0.0010)}{2}(0.7) = 0.00035$$

$$B_p = 2(0.00035)(0.364) = 0.00025 \quad \text{(pinion)}$$

d. Jack-shaft runout at position of 30-tooth gear (backlash due to radial eccentricity of the shaft; probability factor from Table 7):

$$\Delta C_m = \frac{(0.0001)(0.5)}{2} = 0.000025$$

$$B_m = 2(0.000025)(0.364) = 0.00002 \quad \text{(gear)}$$

$$\Delta C_p = \frac{(0.0001)(0.5)(1.0)}{2} = 0.000025$$

$$B_p = 2(0.000025)(0.364) = 0.00002 \quad \text{(gear)}$$

e. Inner-race eccentricity of the ball bearing that supports the 30-tooth gear (backlash due to radial eccentricity of the inner race; from Table 5 for Class 5 bearing):

$$\Delta C_m = \frac{0.0002}{2} = 0.0001$$

$$B_m = 2(0.0001)(0.364) = 0.00007 \quad \text{(gear)}$$

$$\Delta C_p = \frac{(0.0002)(1.0)}{2} = 0.0001$$

$$B_p = 2(0.0001)(0.364) = 0.00007 \quad \text{(gear)}$$

f. Sum of Group C backlash sources (sum of components a through e):

$$\Sigma B_m = 0.00072 + 0.00058 + 0.00036 + 0.00002 + 0.00007 = .00175$$

$$\Sigma B_p = 0.00048 + 0.00040 + 0.00025 + 0.00002 + 0.00007 = 0.00122$$

Phasing factors of 0.7 and 0.7 are used to obtain the net probable backlash for the Group C sources.

$$\Sigma B_{p\ net} = (0.00122)(0.7)(0.7) = 0.00060$$

3. Total linear backlash for mesh A due to Group A, B, and C sources:

$$B_m = 0.00615 + 0.00175 = 0.00790 \text{ in.}$$

$$B_p = 0.00422 + 0.00060 = 0.00482 \text{ in.}$$

4. Total angular backlash of mesh A transferred to the motor pinion (linear backlash, motor pinion pitch radius, and Equation 40):

$r_p$ = motor pinion pitch radius (in.)

n = number of teeth on motor pinion

P = diametral pitch of motor pinion

$\theta_p$ = angular backlash of mesh A referred to motor pinion (rad)

$$r_p = \frac{n}{2P} = \frac{20}{2(48)} = .208 \text{ in.} \qquad (47)$$

$$\theta_{pm} = \frac{B_m}{r_p} = \frac{0.00790}{0.208} = 0.0380 \quad \text{rad} \qquad (48)$$

$$\theta_{pp} = \frac{B_p}{r_p} = \frac{0.00482}{0.208} = 0.0232 \quad \text{rad} \qquad (49)$$

Mesh B and Total Angular Backlash.--The above procedure would be used to calculate the linear backlash of Mesh B. The angular backlash of the mesh would be calculated and referred to the 40-tooth gear; it would then be transferred to the motor pinion by multiplying it by the speed ratio of the motor pinion to the 40-tooth gear; this ratio is 30:20, or 1.5.

The total angular backlash of the system would be the sum of the angular backlash of the 20-tooth motor pinion and the angular backlash of the 40-tooth gear, transferred to the motor pinion. The total angular backlash signifies the angle through which the motor pinion could be rotated while the 60-tooth synchro gear is held stationary.

Methods of Reduction

Antibacklash Gears.--Backlash in a gear train can be eliminated by antibacklash gears. An antibacklash gear, shown in Figure 8, consists of two gear members that are rotationally free relative to one another but are constrained axially. The springs force the two gear sections apart until the tooth space of a full-faced mating gear is completely filled by the teeth of the antibacklash gear; thus, the spring-loaded gear elements adjust themselves automatically to the required tooth thickness. One of the gear members has a hub and is attached firmly to its shaft; the other member is free to rotate on the shaft but is restrained by the springs.

Since the spring tension must be sufficient to produce a spring torque slightly larger than the maximum torque to be transmitted by the gear, the gear is limited to relatively low torque. An antibacklash gear continually eliminates backlash due to all of the above sources; however, it must be spring-loaded to a torque larger than its maximum transmitted torque, which is sometimes difficult to calculate; its total face width is doubled, which doubles its inertia and the inertia of its mating gear; the spring-load increases tooth friction and wear.

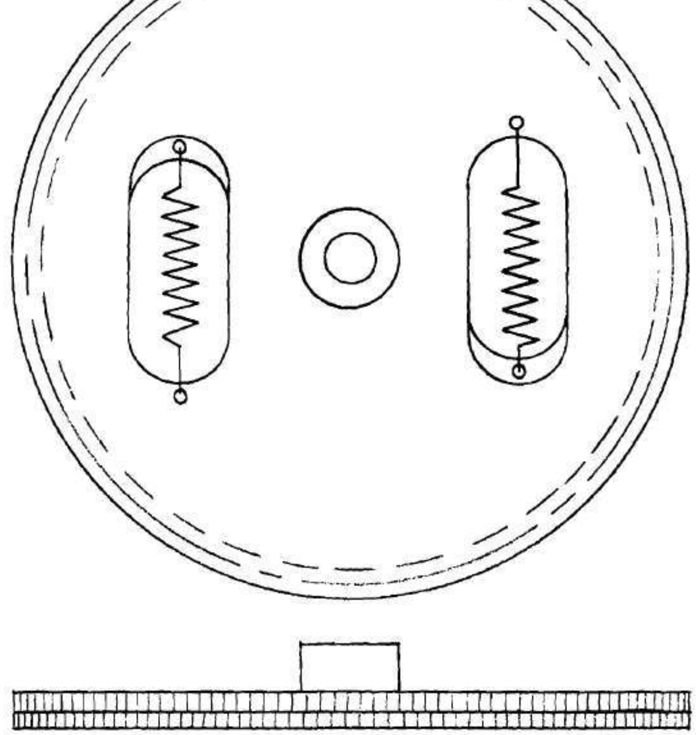

FIG. 8: ANTIBACKLASH GEAR

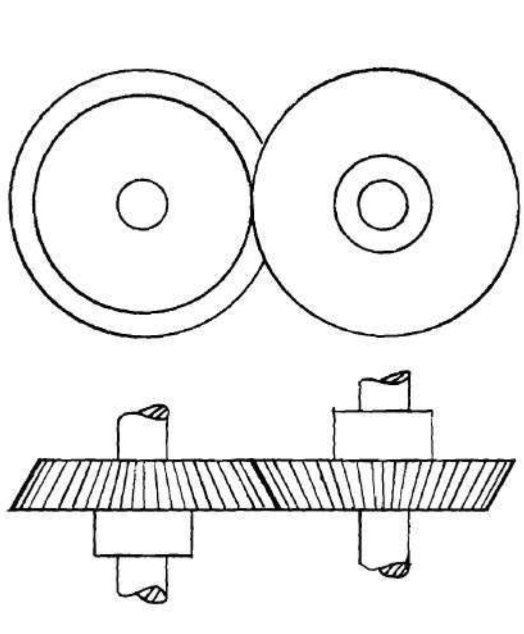

FIG. 9: TAPERED GEARS

Antibacklash gears are most effective in reducing backlash when they are used on the lower speed shafts of a gear train. Meshes on the lower speed shafts contribute the most backlash to the system because of the multiplying effect of the velocity ratio; meshes adjacent to the motor have little effect on the total backlash because of their higher speed.

Adjustable Centers.--Gears with an adjustable center distance can eliminate all backlash except that due to eccentricities (Group C sources). One gear of a mesh is mounted on a shaft which can be adjusted radially so as to vary the center distance between the gear and its mating gear. The gears are meshed, and their center distance is adjusted until there is no backlash when the gears are at their tightest mesh point; then the movable shaft is pinned to prevent further movement.

Tapered Gears.--Tapered gears can eliminate all backlash except that due to eccentricities (Group C sources). A pair of tapered gears, whose pitch surfaces are tangent cones, is shown in Figure 9. Their transmitted torque is limited only by their tooth strength. A fine adjustment of backlash is possible by the axial movement of one of the tapered gears relative to the other.

Pressure Angle.--The system best suited for instrument gears is the 20-Degree, Involute, Fine-Pitch System, which is replacing the 14-1/2 Degree System (19). The advantages of the 20-Degree System are:

1. Higher tooth strength
2. Better tooth surface durability
3. Smoother operation

4. Accurate angular transmission

5. Less tooth sliding

The advantages of the 14-1/2 Degree System are:

1. Lower radial force between meshing gears

2. Lower bearing pressure between meshing tooth surfaces

3. Smaller increase in backlash due to a given increase in gear center distance (See Equations 42 and 43)

It is seen that gears with 14-1/2 degree pressure angles will have slightly less increase in backlash because of a given increase in center distance; however, gears with 20-degree pressure angles are stronger, quieter, and more durable. The advantages of the 20-degree gears make them preferable unless backlash is an extremely critical problem; then the 14-1/2 degree gears should be used.

Diametral Pitch and Precision.--Table 2 and Table 4 show that backlash is reduced by selecting gears of fine diametral pitch and of good precision and class; however, gear tooth strength decreases as the diametral pitch increases, and cost increases as the precision and class are improved. It is evident that a compromise must be reached between backlash, and cost and strength.

# CHAPTER IV

## CONCLUSIONS AND RECOMMENDATIONS

To obtain an optimum design of a servomechanism, the designer should attempt to limit the inertia and backlash of the system to a minimum. Inertia can be reduced by proper selection of overall gear-train ratios and gear-mesh ratios, gear materials, gear shape and size, and electromechanical components. Backlash can be reduced by the use of precision gears, close manufacturing tolerances, proper materials, and antibacklash gears. The minimization of inertia and backlash is essential to obtain an accurate and stable servo system.

# B I B L I O G R A P H Y

## BIBLIOGRAPHY


1. Seeley, F. B., and Ensign, N. E., Analytical Mechanics for Engineers, 3rd ed., New York: John Wiley and Sons, Inc., 1948.

2. Davis, S. A., "Mechanical Components for Automatic Control," Product Engineering, Vol. 25, September 1954, pp. 169-200.

3. Davis, S. A., "Rotating Components for Automatic Control," Product Engineering, Vol. 24, November 1953, pp. 129-60.

4. Burgess, E. G., Jr., "Designing Gear Trains for Minimum Inertia," Product Engineering 1954 Annual Handbook, pp. E 14-20.

5. Hudson, R. G., The Engineers' Manual, 2nd ed., New York: John Wiley and Sons, Inc., 1955.

6. Chestnut, H., and Mayer, R. W., Servomechanisms and Regulating System Design, 1st ed., New York: John Wiley and Sons, Inc., 1955, Vol. II.

7. Harper, J. W., "The Significance of $WK^2$ and How to Calculate It," Product Engineering 1954 Annual Handbook, pp. H 24-31.

8. Holt, J., and Chadbourne, W., "Don't Under-Rate Nylon for Mechanical Components," Product Engineering, Vol. 24, June 1953, pp. 203-11.

9. Lurie, Robert, "Nylon -- Its Properties and Applications," Materials and Methods, Vol. 36, July 1952, pp. 79-83.

10. Martin, L. D., "How Inserts Improve Nylon Gears," Tool Engineer, Vol. 33, December 1954, pp. 79-84.

11. Canick, L. N., "Optimum Gear Train Ratios for Instrument Servomechanisms," Product Engineering, Vol. 22, July 1951, pp. 108-9.

12. Fuchs, A. M., "Motor and Gear Train Selection for Automatic Control Systems," Product Engineering, Vol. 24, March 1953, pp. 128-33.

13. Perkins, G. S., "Analysis of Servomechanisms for Instrument and Power Drives," Product Engineering, Vol. 17, April 1946, pp. 332-4.

14. Lauer, H., Lesnick, R., and Matson, L., Servomechanism Fundamentals, 1st ed., New York: McGraw-Hill Book Company, Inc., 1947.

15. Ahrendt, W. R., Servomechanism Practice, 1st ed., New York: McGraw-Hill Book Company, Inc., 1954.

16. Thaler, G. J., and Brown, R. G., Servomechanism Analysis, 1st ed., New York: McGraw-Hill Book Company, Inc., 1953.

17. Spotts, M. F., Design of Machine Elements, 1st ed., New York: Prentice-Hall, Inc., 1948.

18. Michalec, G. W., "Precision Gearing," Machine Design, Vol. 27, January 1955, pp. 154-63.

19. Martin, L. D., "Instrument Gears......Which Pressure Angle," Machine Design, Vol. 26, February 1954, pp. 129-35.

20. Thoen, R. L., "Gear Eccentricity......How to Predict Its Effect on Angular Displacement," Product Engineering, Vol. 22, September 1951, pp. 155-8.

21. Martin, L. D., "Designing Instrument Gear Trains," Product Engineering, Vol. 24, May 1953, pp. 120-4.

22. Dean, P. M., Jr., "Effects of Size on Gear Design Calculations," Product Engineering, Vol. 25, April 1954, pp. 129-40.

23. Martin, L. D., "Designing Instrument Gearing," Machine Design, Vol. 20, April 1948, pp. 163-8.

24. Michalec, G. W., "Precision Gearing," Machine Design, Vol. 27, February 1955, pp. 200-9.

25. Lofgren, K. E., "Zero Backlash Arrangements for Gear Trains and Drives," Product Engineering, Vol. 18, July 1947, pp. 150-2.